A NOMOGRAPHIC METHOD FOR PREDICTING THE BEHAVIOR OF A PETROLEUM RESERVOIR

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B.S., University of Washington, 1949

Submitted to the Graduate School of the University of Pittsburgh in partial fulfillment of the requirements for the degree of Master of Science

Pittsburgh, Pennsylvania

1956

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I. INTRODUCTION

The Material Balance Method of calculating the behavior of petroleum reservoirs is one of the petroleum engineer's most important tools. Introduced in 1936, the Material Balance has undergone many improvements intended to simplify the mathematical techniques of handling the complex physical relationships involved. In spite of all advances in technique, the method remains unwieldy and laborious.

The basic concept of the Material Balance applies the Law of Conservation of Mass to the process of producing petroleum from an underground reservoir. The amount of hydrocarbons originally present in the reservoir must equal the amount produced plus the amount remaining in the ground at any given time during the production life of the reservoir. As normally used, the Material Balance is written volumetrically. It relates the volumes of oil and gas (produced and remaining) by means of their physical characteristics (gas volume, gas solubility and oil shrinkage) at the pressure prevailing in the reservoir.

The Material Balance has been most widely used for the solution of two general problems: the estimation of original oil in place and the prediction of future production performance of a reservoir. The reservoir discussed in this paper will be of the Solution Gas Drive Type, with fixed reservoir volume, and having no gas cap and no water encroachment into the reservoir.

A. Estimation of the Original Oil in Place

The quantity of oil and gas originally present in a reservoir may be estimated by observing the drop in reservoir pressure during the production of a certain amount of fluids, and relating these data to the laboratory-measured behavior of the reservoir fluids caused by pressure changes. The volume of reservoir fluids (with known shrinkage and solubility characteristics) which will undergo a certain pressure change because of the removal of a given quantity of these fluids is a fixed quantity. Thus, the drop in reservoir pressure caused by a given amount of oil and gas being produced defines the volume of these fluids originally present.

B. Prediction of the Future Production Performance of a Reservoir

For any given pressure assumed to prevail in the reservoir at some future time, the volumes of oil and gas produced (when related to their physical characteristics at that pressure) constitute a unique solution to the equation balancing the original volume of fluid with the volumes produced and remaining. An additional concept is involved however. The permeability of the porous reservoir medium to oil and to gas changes as production proceeds. Thus, as the saturation of liquid in the reservoir decreases as oil is produced, the relative amounts of oil and gas which flow to the well will change. This phenomenon cannot be directly related to the Material Balance itself, yet it determines the relative quantities of oil and gas which will be produced. The relationship between the relative permeability to oil and gas and the liquid saturation in the reservoir must be known or assumed, since it must be considered in arriving at the amounts of oil and gas to be produced. These amounts of oil and gas can then be used to predict a solution to the Material Balance at some assumed future point of pressure and production.

It will be the purpose of this thesis to develop a graphical means for predicting the performance of a depletion drive reservoir based on a modification of the Material Balance equation.

II. REVIEW OF THE LITERATURE

The use of the Material Balance for estimating oil in place was developed by Schilthuis in 1936.[1] His equation (shown here for a solution gas drive reservoir without gas cap, water encroachment or water production)

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o}^{*} \tag{1}$$

still remains the most used basic form of the Material Balance.

At about the same time Katz[2] proposed a tabular method for evaluating oil in place. This method was later shown by Pirson[3] to be equivalent to the Schilthuis method.

The use of the Material Balance Equation for prediction of future reservoir performance was proposed considerably later. Babson[4] developed a trial and error solution based on the Material Balance Equation and the Instantaneous Gas-Oil-Ratio Equation. His method was very cumbersome and required a great deal of computation. It is rarely used today. In the same year, 1944, Tarner[5] proposed a solution to the problem which was considerably simpler to use.

Muskat[6] developed a form of the Material Balance expressed as a differential equation and applied it to prediction of depletion drive reservoir performance.

The most widely used of the Material Balance prediction techniques is the method using a trial and error solution of Schilthuis' Equation[7, 8]. This method requires the simultaneous satisfaction of three equations:

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[1] References are listed in the Bibliography.
*Nomenclature shown in Appendix I.

1. The Material Balance Equation

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o} \qquad (1)$$

2. The Instantaneous Gas-Oil-Ratio Equation

$$r_i = \frac{\mu_o}{\mu_g}\frac{\beta}{v}\frac{K_g}{K_o} + s \qquad (2)$$

3. The Liquid Saturation Equation

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n}{\beta_o}\right)\beta \qquad (3)$$

The trial and error solution may be conducted in the following manner:

1. Assume a certain drop in reservoir pressure
2. Estimate the volume of oil which would be produced during this pressure drop
3. Using this estimated value of oil production, solve the Liquid Saturation Equation, (3), and obtain the corresponding value of $\frac{K_g}{K_o}$ from the known or assumed $\frac{K_g}{K_o}$ vs. S_L relationship.
4. Solve Equation (2) for the value of Instantaneous Gas-Oil-Ratio which would obtain at the assumed pressure.
5. Using the assumed and estimated values of oil production, gas-oil-ratio and fluid characteristic solve the Material Balance Equation. If a balance is obtained, the assumed values were correct. If the equation does not balance, a new value of oil production or pressure is estimated and another trial is made.

This process is repeated until the equations are satisfied at sufficient pressure points to define the future performance of the reservoir.

The use of the Schilthuis Equation, while more flexible in the handling of such conditions as water encroachment and gas cap expansion, is extremely tedious. For the simplest reservoirs, twenty-five columns of calculations must be used, and five or six trials at each of seven or eight pressure points is not unusual. All this must be performed with a calculating machine. Despite this, it is still the most widely used mathematical prediction technique.

In January, 1955, a prediction method based on the Schilthuis method, but greatly simplified, was published by Tracy.[9] The Tracy method is the single biggest advance in the use of the Material Balance Equation. This method will be described for the case of a depletion drive reservoir without gas cap or water encroachment.

Tracy took the Schilthuis Equation

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o} \tag{1}$$

and substituted values for u, u_o and r_n as follows:

$$u = \beta + (s_o - s)\ v \tag{4}$$

$$u_o = \beta_o \tag{5}$$

$$r_n = \frac{G}{n} \tag{6}$$

This substitution resulted in:

$$N = \frac{n\left(\frac{\beta}{v} - s\right) + G}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \tag{7}$$

which may be expanded to:

$$N = \frac{n\left(\frac{\beta}{v} - s\right)}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} + \frac{G}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (8)$$

Thus, n and G are multiplied by coefficients which are functions of pressure only. These coefficients are:

$$\phi_n = \frac{\left(\frac{\beta}{v} - s\right)}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (9)$$

$$\phi_g = \frac{1}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (10)$$

Equation (8) then reduces to:

$$N = n\,\phi_n + G\,\phi_g \qquad (11)$$

In this form, the equation may be used to estimate original oil in place.

Tracy's prediction method uses this equation in two forms with N taken as equal to one barrel of stock and oil.

1. As the Prediction Equation:

$$\Delta n = \frac{1 - (n_{i-1}\,\phi_n + G_{i-1}\,\phi_g)}{\phi_n + \frac{r_i + r_{i-1}}{2}\,\phi_g} \qquad (12)$$

2. As the Material Balance Equation:

$$n_i\,\phi_n + G_i\,\phi_g = 1 \qquad (13)$$

Equations (12) and (13) are used in conjunction with Gas-Oil-Ratio Equation:

$$r_i = \frac{\mu_o}{\mu_g} \frac{\beta}{V} \frac{K_g}{K_o} + s \qquad (2)$$

and Liquid Saturation Equation:

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n_i}{\beta_o}\right)\beta \qquad (3)$$

Equations (2) and (3) being linked by knowledge of the $\frac{K_g}{K_o}$ vs. S_L relationship.

The use of Tracy's Prediction Method calls for the following procedure:

1. Starting with a point in production history (with known values of n_i, G_i ϕ, r_i for a given pressure), assume a new lower pressure point.

2. Estimate an instantaneous gas-oil ratio for this pressure

3. Using this pressure and gas-oil ratio in the Prediction Equation (12), compute a value for Δn.

4. Add this value of Δn to $n_i - 1$ (n_i from the previous pressure) to get the value of n_i for the new assumed pressure.

5. Solve the Liquid Saturation Equation, (3), using this value of n_i and obtain a value for $\frac{K_g}{K_o}$.

6. Using the $\frac{K_g}{K_o}$ value, solve the Gas-Oil Ratio Equation. This value of Instantaneous Gas-Oil Ratio should equal that estimated in step 2. If it does not, use the value of r_i found in step 6 as the estimated value in step 2 and re-do steps 2 through 6. When the values of Instantaneous Gas-Oil Ratio in steps 2 and 6 are equal, substitute the values of n_i and

G_i into the Material Balance Equation (13) and solve. The sum of $n_i \phi_n + G_i \phi_g$ should be 1.00 ± 0.02.

The number of trials required is cut down to two or three in Tracy's method because Instantaneous Gas-Oil Ratio - a relatively insensitive factor - is estimated. This cannot be done in the Schilthuis Equation as the cumulative value of gas-oil ratio, rather than the instantaneous value, is used in the equation. The cumulative gas-oil ratio value cannot be estimated with any degree of accuracy.

Sturdivant,[10,11,12] in a series of three articles, expanded upon the use of the Tracy method developing a formula to be used in the event of gas reinjection into the reservoir. It was also shown[11] that accuracy to three significant figures in the calculation was sufficient.

III. STATEMENT OF THE PROBLEM

This investigation was conducted with the object of developing a graphical method for solution of the Performance Prediction problem. It was felt that the Prediction Equation developed by Tracy, with its separation of the pressure factors into coefficients, would lend itself to such treatment. In the Schilthuis Equation, the complexity of form appeared to preclude any simple graphical treatment.

Of the various graphical problem solving methods available, nomography appeared as the best for trial and error solutions. The forms of the equations indicated that nomographs of reasonable simplicity could be constructed to solve them. For trial and error solutions, it was felt that the nomographs should be simple enough that the operator could see at a glance how much a given change in one factor would affect the other factors in the problem. Also, with simple, one-setting nomographs, the operator can readily enter the diagram with the dependent variable (or answer) and work backwards to find the value of independent variable (or normal input). This is of value on trial and error solutions. For these reasons, as well as for ease of construction, it was desirable that the nomographs be of the type in which a single line crosses all scales, obtaining the answer in a single setting.

There are a number of nomographs in the literature concerned with various aspects of the Material Balance problem[13,14,15,16]. All of them are quite complex, requiring entry with several individual pressure variables, and requiring several settings of a line before a solution is obtained. These nomographs are general in application and could be used on reservoirs having the same ranges of variables as the nomographs.

By constructing nomographs for a specific reservoir, all of the fluid characteristics (which are functions of reservoir pressure) and combinations of them, can be treated as a single, or recurring, variable of pressure. This reduces the number of variables to the point where simple, single-setting nomographs can be constructed. Construction of the nomographs for a single reservoir allows selection of optimum scale lengths to specially handle the ranges of the variables for that reservoir. More generalized nomographs would require the ranges of the scales to be broad enough to fit all or several reservoirs. This would result in less accuracy for any particular reservoir.

IV. METHOD OF INVESTIGATION

The four equations used in the Tracy method of prediction were separately investigated to find the forms most likely to yield the simplest nomographs. These equations, given previously as Equations (12), (13), (2) and (3) will be considered separately.

A. The Prediction Equation

This equation was given[17] as:

$$\Delta n = \frac{1 - (n_{i-1}\,\phi_n + G_{i-1}\,\phi_g)}{\phi_n + r_a\,\phi_g} \tag{14}$$

where

$$r_a = \frac{r_i + r_{i-1}}{2} \tag{15}$$

It is readily rearranged into this form:

$$(n_{i-1} + \Delta n)\,\phi_n + (G_{i-1} + r_a\,\Delta n)\,\phi_g = 1 \tag{16}$$

In this form, it is seen to be equivalent to the Material Balance Equation as:

$$n_{i-1} + \Delta n = n_i \tag{17}$$

and

$$G_{i-1} + r_a\,\Delta n = G_i \tag{18}$$

If these values are substituted into Equation (16), the result is the Material Balance Equation:

$$n_i\,\phi_n + G_i\,\phi_g = 1 \tag{13}$$

Returning to the rearranged form of the Prediction Equation:

$$(n_{i-1} + \Delta n)\,\phi_n + (G_{i-1} + r_a\,\Delta n)\,\phi_g = 1 \tag{16}$$

it is noted that there are four variables:

$(n_{i-1} + \Delta n)$, a function of oil production

Let this be f_1 (u)

$(G_{i-1} + r_a \Delta n)$, a function of the gas produced with the above amount of oil

Let this be f_2 (v)

ϕ_n, a function of reservoir pressure

Let this be f_3 (W)

ϕ_g, a different function of reservoir pressure

Let this be f_4 (W)

Thus, Equation (16) can be written as:

$$f_1 (u) \times f_3 (W) + f_2 (v) \cdot f_4 (W) = 1 \tag{17}$$

Note that the pressure, represented here by "W" is a recurrent variable.

Divide Equation (17) by f_3 (W), then:

$$f_1 (u) + f_2 (v) \cdot \frac{f_4 (W)}{f_3 (W)} = \frac{1}{f_3 (W)} \tag{18}$$

Now let

$$\frac{f_4 (W)}{f_3 (W)} = f_5 (W), \text{ a different function of pressure}$$

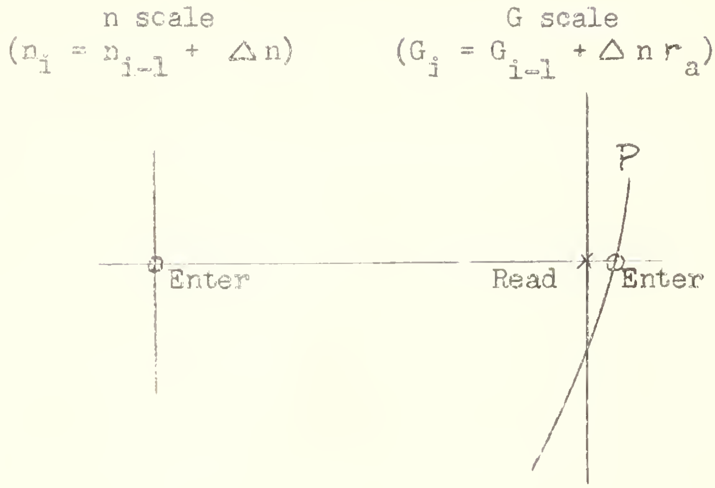
and

$$\frac{1}{f_3 (W)} = f_6 (W), \text{ another function of pressure.}$$

Equation (18) now becomes:

$$f_1 (u) + f_2 (v) \times f_5 (W) = f_6 (W) \tag{19}$$

This form is capable of representation by a simple nomograph having the following configuration:



The geometric proof for this nomograph is given in Appendix IV.

Now, evaluating the pressure functions f_5 (W) and f_6 (W) results in:

$$f_5 (W) = \frac{f_4 (W)}{f_3 (W)} = \frac{\phi_g}{\phi_n} = \frac{1}{\left(\frac{\beta}{v} - s\right)} \tag{20}$$

$$f_6 (W) = \frac{1}{\phi_n} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \tag{21}$$

These values, when substituted into the Prediction Equation, give:

$$(n_{i-1} + \Delta n) + \frac{(G_{i-1} + r_a \Delta n)}{\left(\frac{\beta}{v} - s\right)} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \tag{22}$$

Equation (22) is the form of Tracy's Prediction Equation which is solved by the Prediction Equation Nomograph.

B. The Material Balance Equation

This equation is equivalent to the Prediction Equation and can use the same nomograph. One solution of the nomograph serves to satisfy both the Prediction Equation and the Material Balance Equation.

Placed in the form used to construct the nomograph, the Material Balance Equation becomes:

$$n_i + \frac{G_i}{\left(\frac{\beta}{v} - s\right)} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \qquad (23)$$

Equation (23) is solved by the Prediction Equation nomograph.

B. [illegible]

This [illegible] Appendix [illegible]

the same [illegible]

the Pacific [illegible]

[illegible]

Belgian [illegible]

C. The Liquid Saturation Equation

The Liquid Saturation Equation is used to obtain a value of $\frac{K_g}{K_o}$ from a known and extrapolated relationship between S_L and $\frac{K_g}{K_o}$.

In the equation

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n_i}{\beta_o}\right)\beta \tag{3}$$

The values of S_W and β_o remain constant.

Let S_W be K_1 and let $\left(\frac{1 - S_W}{\beta_o}\right)$ be K_2.

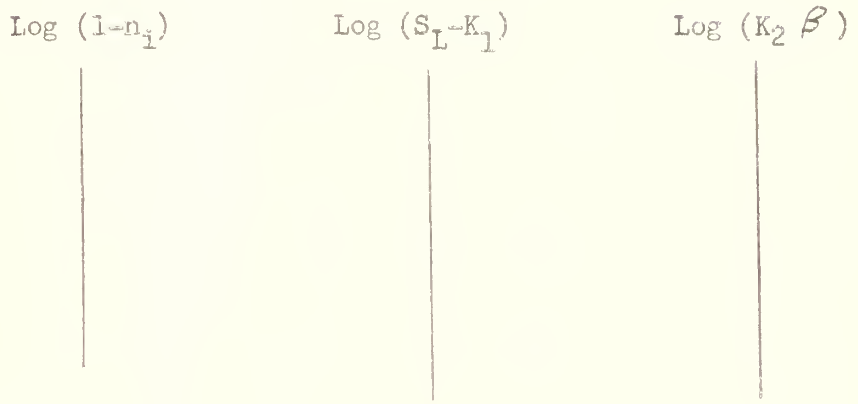
The equation can then be rearranged:

$$S_L - K_1 = (K_2)(\beta)(1 - n_i) \tag{24}$$

or

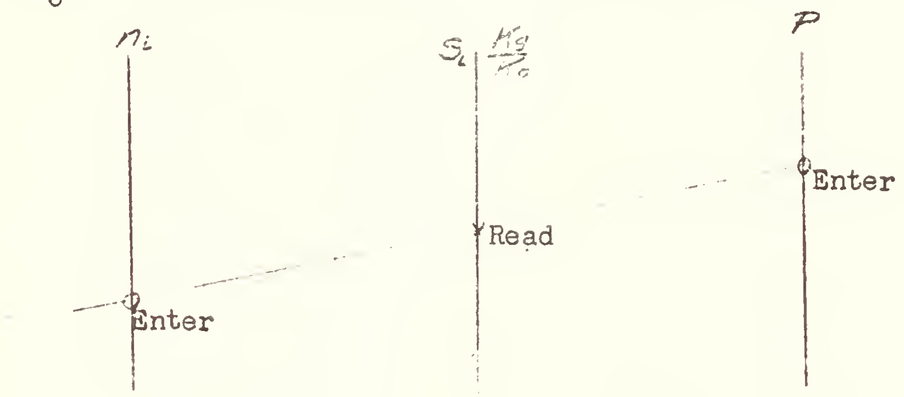
$$\text{Log}(S_L - K_1) = \text{Log}\, K_2\beta + \text{Log}(1-n_i) \tag{25}$$

Equation (25) was used to construct a parallel line nomograph with logarithmic scales having the following form:*



*For proof of the geometry of this nomograph, see Appendix IV.

Log $(1-n_i)$ is a simple function of n_i and the left scale was therefore graduated directly in units of n_i. Similarly, β , and therefore Log $K_2\beta$ is a function of pressure, so that the right scale was graduated in units of pressure. The center scale was graduated on one side in units of S_L and by use of the known $\frac{K_g}{K_o}$ vs. S_L relationship, the other side was graduated in units of $\frac{K_g}{K_o}$. The finished nomogram appeared as follows:



D. The Gas-Oil Ratio Equation

In the instantaneous Gas-Oil Ratio Equation

$$r_i = \frac{\mu_o}{\mu_g} \frac{\beta}{v} \frac{K_g}{K_o} + s \qquad (2)$$

$\frac{\mu_o}{\mu_g}$, β, v, and s are specific functions of the reservoir pressure. By combining all of these but s into a single function of pressure, the equation was rearranged to:

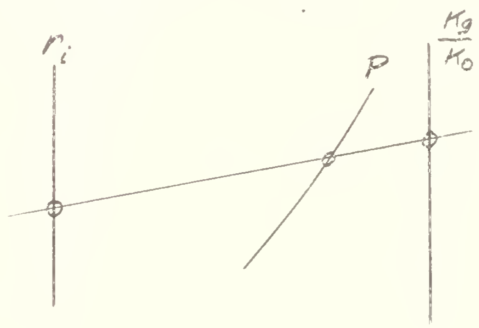
$$r_i - \frac{K_g}{K_o} \times (F) = s \qquad (26)$$

This form is identical to

$$f_1 (u) - f_2(v) \cdot f_3 (W) = f_4 (W) \qquad (27)$$

where f_3 (W) and f_4 (W) are different functions of the recurrent variable, reservoir pressure.

A nomograph, similar in theory* to the Prediction Equation nomograph was constructed. The finished form appeared as follows:



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*See Appendix IV for geometric proof.

E. Average Gas-Oil Ratio Equation

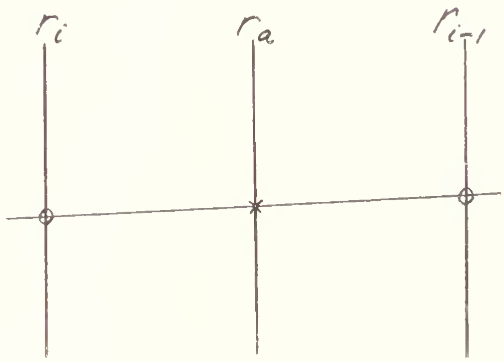
It was found desirable to construct a very simple nomograph to solve the equation

$$\frac{r_i + r_{i-1}}{2} = r_a \qquad (15)$$

The equation was rearranged to:

$$r_i + r_{i-1} = 2\, r_a$$

A nomograph with uniform parallel scales set equal distances apart was constructed.



Entry can be made on any two scales and a value read on the third scale.

V. USE OF THE NOMOGRAPHS TO PREDICT THE PERFORMANCE OF A SPECIFIC RESERVOIR

To test the application of the nomographic prediction method, an actual reservoir was selected and nomographs constructed using the data available concerning this reservoir. The particular reservoir chosen was a mid-continent limestone reservoir, which is still in the early stages of production. For this reason, it will be called Reservoir "X." The basic data available on Reservoir "X" are listed in Appendix V. In addition, two sets of prediction curves (Pressure and Gas-Oil Ratio Vs. Cumulative Production) were available for comparison with the results obtained in this investigation by the nomographic method. One of these sets of prediction curves was obtained by trial and error use of the Schilthuis Equation. The other was obtained by using Tracy's method.

The insensitivity of instantaneous gas-oil ratio values over a range of incremental oil production values was mentioned in connection with the Tracy method. The relative positions of the n, G and P scales on the Prediction Nomograph show this feature graphically. The pressure scale is very close to the gas production scale. For this reason, it is best to enter the nomogram with an assumed value of oil production, rather than to assume a gas-oil ratio as in the tabulated Tracy method.

The following approach was found to give the easiest solution using the nomograms:

1. Set up a tabulation sheet as follows:

(0)	P	1209 (A known point of Production History)	1100	900
(1)	Δn		0.00773	0.01248
(2)	n_{i-1}		0.0319	0.03963
(3)	n_i	0.0319	0.03963	0.05211
(4)	$G_{i-1} + \Delta n\ r_a$	46.75	65.5	108.3
(5)	G_{i-1}		46.75	65.5
(6)	$r_a \Delta n$		18.75	42.8
(7)	r_a		2425	3430
(8)	r_{i-1}		2400	2450
(9)	r_i	2400	2450	4420
(10)	$\frac{K_g}{K_o}$		0.0362	0.085
(11)	r_i	(From G.O.R. Equation)	2450	4420
(12)	$(n_i)\ (1.8 \times 10^7)$		713.0	937.5
(13)	+575,000		575.0	575.0
(14)	Cumulative Production (in 10^3 bbls.)		1288.0	1512.5

2. Select a point of actual production history, knowing pressure, oil production from the bubble point to that pressure, n_i, expressed as a fraction of original oil in place, gas production from the bubble point to that pressure expressed as a function of original oil in place, and instantaneous gas-oil ratio, r_i.

3. Assume a new pressure, 100-200 psi lower than the historical point used. The n_i, G_i and r_i of the historical pressure point now become n_{i-1}, G_{i-1}, and r_{i-1} in the above tabulation.

4. Estimate an increment of oil production, Δn. Enter this on

line one of the tabulation. Add to it the n_{i-1}. The sum is n_i.

5. Using this value of n_i at the assumed pressure, enter the prediction nomograph on the n and P scales and read the value of $G_{i-1} + \Delta n\ r_a$ on the G scale. Enter this value on line 4 of the tabular sheet and subtract the G_{i-1} from it. The difference is $\Delta n\ r_a$.

6. With a slide rule, divide the value of $\Delta n\ r_a$, (line 6) by Δn (line 1) to get r_a (line 7).

7. Enter the average gas-oil ratio nomograph with r_{i-1} and r_a and find r_i (line 9).

8. Using n_i (line 3) and Pressure, find $\frac{K_g}{K_o}$ from the liquid nomograph and enter this value on line 10.

9. Enter the gas-oil ratio nomograph with $\frac{K_g}{K_o}$ and Pressure and find the r_i value based on the gas-oil ratio equation. Place on line 11.

10. Compare lines 9 and 11 which will be equal if the assumed value of Δn was correct.

Note that as assumed values of Δn are increased, resulting values of r_i on line 9 (from prediction nomograph) decrease sharply, whereas values of r_i on line 11 (from gas-oil ratio nomograph increase slowly. Thus, if the r_i of line 11 is less than that of line 9, a slight increase of Δn will bring them together. If the two values of r_i are within a few hundred of each other, set the $\Delta n\ r_a$ value from line 6 on the slide rule "D" scale, place the value of r_a (line 7) opposite it on the "C" scale, and read a nearly-correct value for Δn under the index on the "D" scale.

If this new value of Δn causes a radical change in n_i, it may be necessary to enter the nomograph for a new $G_{i-1} + \Delta n\ r_a$. However, from the scale positions, it can be seen that $G_{i-1} + \Delta n\ r_a$ is very insensitive to changes in Δn at most pressures.

11. If the new value of Δn caused more than a slight change in n_i, re-enter the liquid saturation and gas-oil ratio nomograms to obtain a corrected r_i for line 11.

12. Lines 9 and 11 should now be within perhaps 50 or 60 of each other. Alter Δn slightly to bring line 9 in exact agreement with line 10. Add the corrected Δn to n_{i-1}. The result is n_i.

13. Multiply n_i by the volume of original oil in place for line 12 and add line 13, the production from the original pressure to the bubble point. Line 14 is the predicted cumulative production at the assumed pressure.

14. Assume a new pressure about 200 psi lower and repeat the process.

Special care in the averaging of r_i and r_{i-1} must be taken when computing at about the peak of the Gas-Oil Ratio Vs. Production Curve, as any averaging process assumes local linearity of a curve. It is best to use smaller pressure increments in this region.

Using the method described above, about two hours were required to calculate the predicted performance using seven pressure steps.

An alternate method was developed which takes about the same time to use, but which may be a little more accurate:

1. Use the same tabulation sheet as shown on page 22 above. In addition, a large "scratch graph sheet" of squared paper (10 x 10 to the inch was used) and a sheet of scratch paper are necessary.

2. Same as step 2 above.

3. Same as step 3 above.

4. Estimate three values of Δn quite close together. Jot on scratch sheet or carry mentally.

5. Enter the prediction nomogram with the middle value of Δn and read a value of $G_{i-1} + \Delta n\ r_a$. (It will be seen that the change in $G_{i-1} + \Delta n\ r_a$ over the three values of Δn is so small as to be unreadable.)

6. Subtract G_{i-1} from $G_{i-1} + \Delta n\ r_a$. Divide $\Delta n\ r_a$ by each of the three values of Δn from step 4 obtaining three values of r_a.

7. On the scratch graph sheet, plot the three values of r_a vs. the respective Δn values.

8. Add the three values of Δn to n_{i-1}. Use the resulting 3 values of n_i in the liquid saturation and gas-oil ratio nomographs to obtain three values of r_i.

9. Using the average gas-oil ratio nomograph and entering with r_{i-1} and the three r_i values of step 8, find three values of r_a.

10. Plot these values against Δn values on the scratch graph sheet.

11. Where the lines of step 7 and step 10 cross, read the correct values of Δn and r_a. Enter these values on the tabular sheet and check the computation on the nomographs, comparing tabulation lines nine and 11.

12. Make minor adjustments of Δn to match line 9 to line 11.

13. Same as step 13 above.

14. Same as step 14 above.

Figure 1 is a graph comparing the results obtained by the nomographic method with those obtained by trial and error solution of the Schilthuis Equation and by Tracy's method. The results in general are in good agreement. The higher gas-oil ratio values shown on the Schilthuis method curve are thought to have resulted from use of different values of "v" than were used in the Tracy method and in the construction of the nomographs. The Schilthuis method curve was taken from an engineering consultant's report on Reservoir "X".

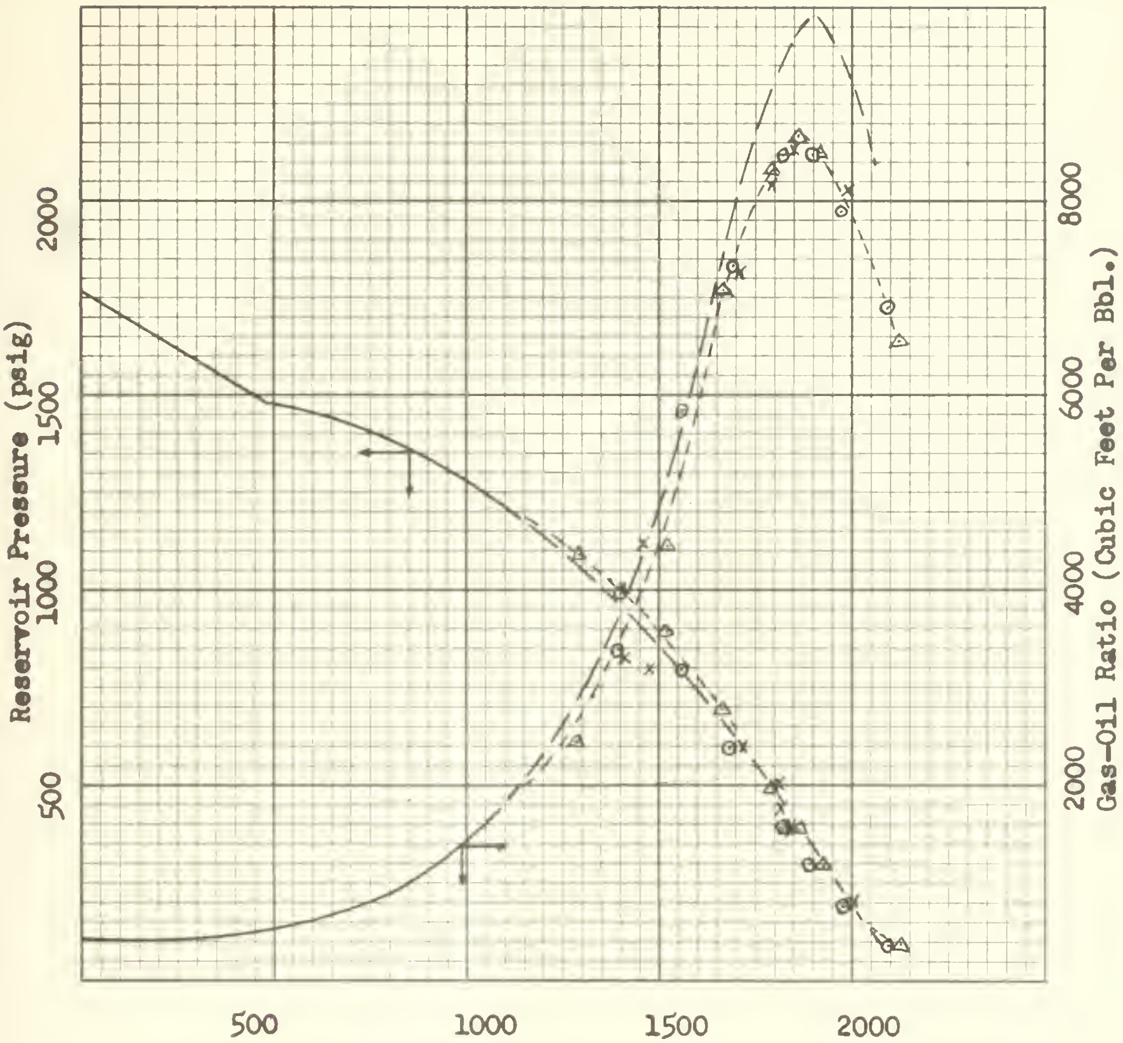


Figure 1

Comparison of Reservoir Performance Prediction Results

Legend

——— Production History

- - - - - Mean of Nomographic Predictions

— — — Prediction by Schilthuis Equation

⊙ Prediction by Nomographic Method First Described

△ Prediction by Alternate Nomographic Method

× Prediction by Tracy's Method

VI. SUMMARY AND CONCLUSIONS

This investigation had as its object the development of a graphical technique for predicting production performance of a petroleum reservoir. A graphical technique involving the use of nomographs based on the Tracy method was worked out. Using this method nomographs were constructed to predict the performance of a specific actual reservoir called Reservoir "X." The performance of Reservoir "X" under depletion drive was predicted using these nomographs. The results, compared with the results obtained by other methods, show that the use of nomographs constructed for a specific reservoir gives a forecast of reasonable accuracy.

The total time involved in nomograph construction and in solving the prediction problem with these nomographs proved to be about the same as that required to solve a reservoir problem by the Tracy method. However, reservoir performance predictions are often repeated, starting with a later point in production history, as the field develops. With this in mind, it is believed that the nomographic method would save considerable calculation. A repeated prediction could be run in about two hours using the same nomographs. A repeated calculation, even by the Tracy method, would take considerably longer.

As the production history of the field develops, it may be found that the basic field data relating relative permeability ratio and saturation were extrapolated in error. In this event, new and more correct values of $\frac{K_g}{K_o}$ may be marked on the center scale of the liquid saturation nomograph without affecting its accuracy for subsequent use.

The limited time available precluded investigating the application of this technique to a reservoir undergoing gas reinjection. It is almost

certain that a set of reasonably simple nomographs could be constructed to handle predictions under various conditions of gas reinjection.

The time available was also too short to allow the method to be applied to reservoirs of differing characteristics. In particular, it should be tried on a sandstone reservoir, on a reservoir with a high initial pressure, and a reservoir with a low initial pressure.

APPENDIX I

Nomenclature

F = $\frac{\mu_o}{\mu_g} \cdot \frac{\beta}{v}$

G = Cumulative gas production, standard cubic feet

G_i = G at reservoir pressure being considered

G_{i-1} = G at previously considered reservoir pressure

$\frac{K_g}{K_o}$ = Relative permeability ratio, a dimensionless number

N = Total original oil in place, in stock tank barrels

n = Cumulative oil production, expressed either in stock tank barrels (or as a fraction, when N is taken as equal to one barrel)

n_i = The value of n at the reservoir pressure being considered

n_{i-1} = The value of n at the last previously considered reservoir pressure

r_a = Average gas-oil ratio, standard cubic feet per barrel of stock tank oil

r_i = Instantaneous gas-oil ratio at reservoir pressure being considered standard cubic feet per barrel of stock tank oil

r_{i-1} = Instantaneous gas-oil ratio at last previous reservoir pressure, standard cubic feet per barrel of stock tank oil

r_n = Cumulative gas-oil ratio, or the ratio of total standard cubic feet of gas produced to total barrels of stock tank oil produced

S_L = Total liquid saturation in the reservoir

S_o = Oil saturation in the reservoir

S_W = Saturation of connate water in the reservoir

s = Solution gas-oil ratio, or the solubility of gas in crude oil at reservoir pressure and temperature, cubic feet per barrel

s_o = Value of s at original reservoir pressure

u = Volume in the reservoir occupied by one barrel of stock tank oil plus all the gas originally in solution in that oil, or the two-phase formation volume factor

u_o = Value of u at original reservoir pressure $u_o = \beta_o$

v = Barrels of free gas space occupied in the reservoir by one standard cubic foot of gas

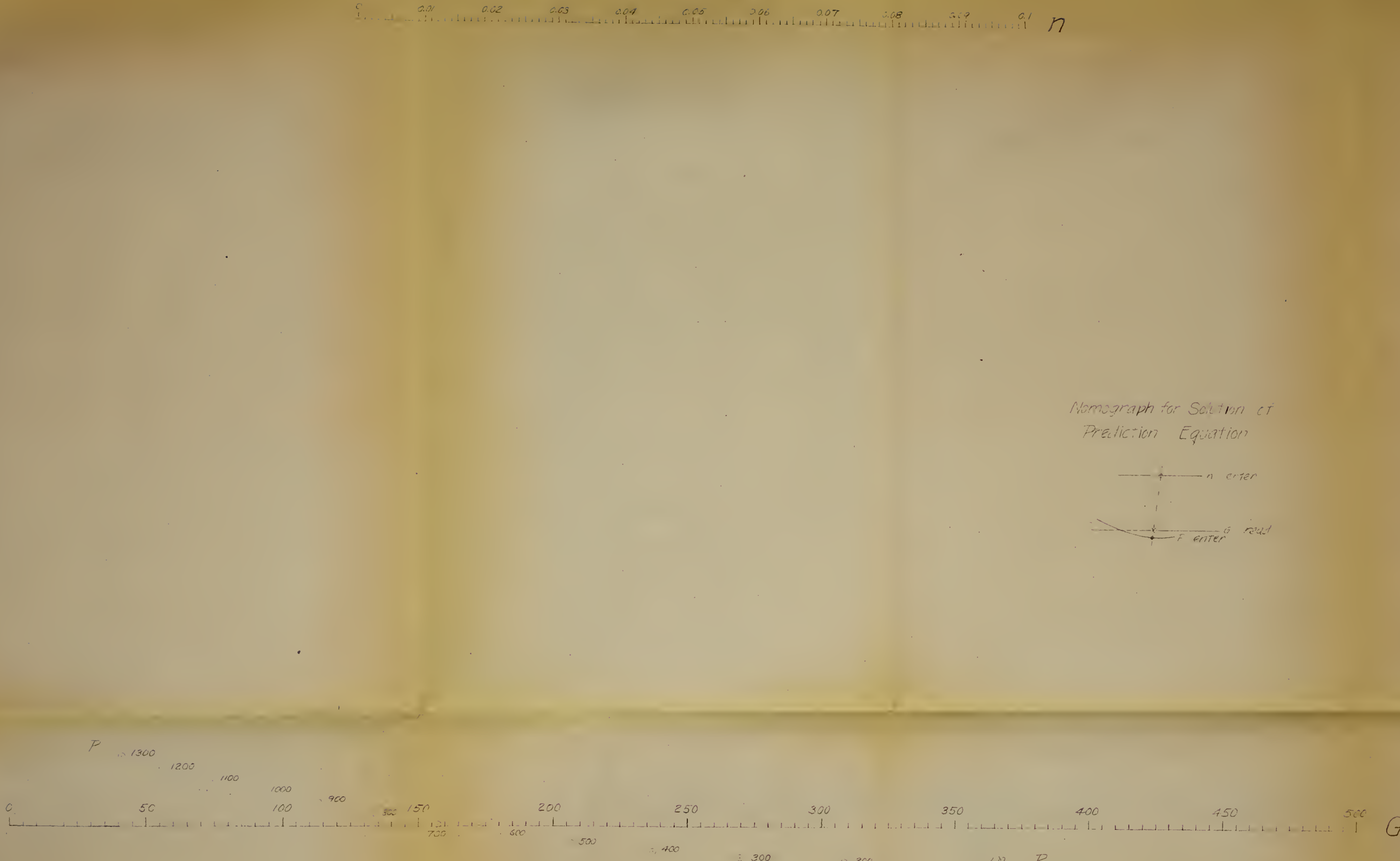
β = Formation volume factor, or the volume occupied at reservoir conditions by one barrel of stock tank oil plus its dissolved gas

β_o = Value of β at original reservoir pressure

Δn = Increment of oil production between two reservoir pressure steps

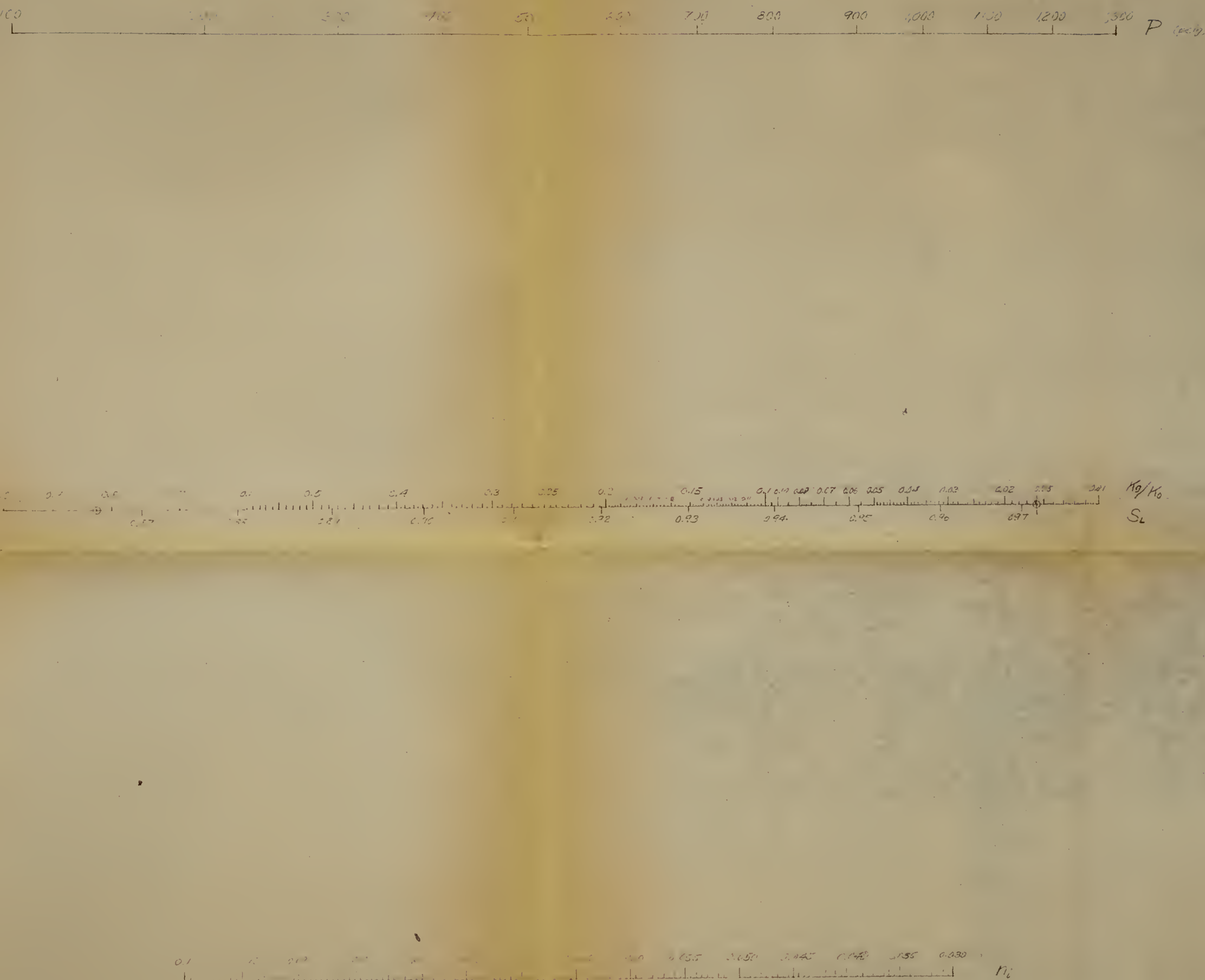
μ_o = Viscosity of reservoir oil at reservoir conditions, centipoises

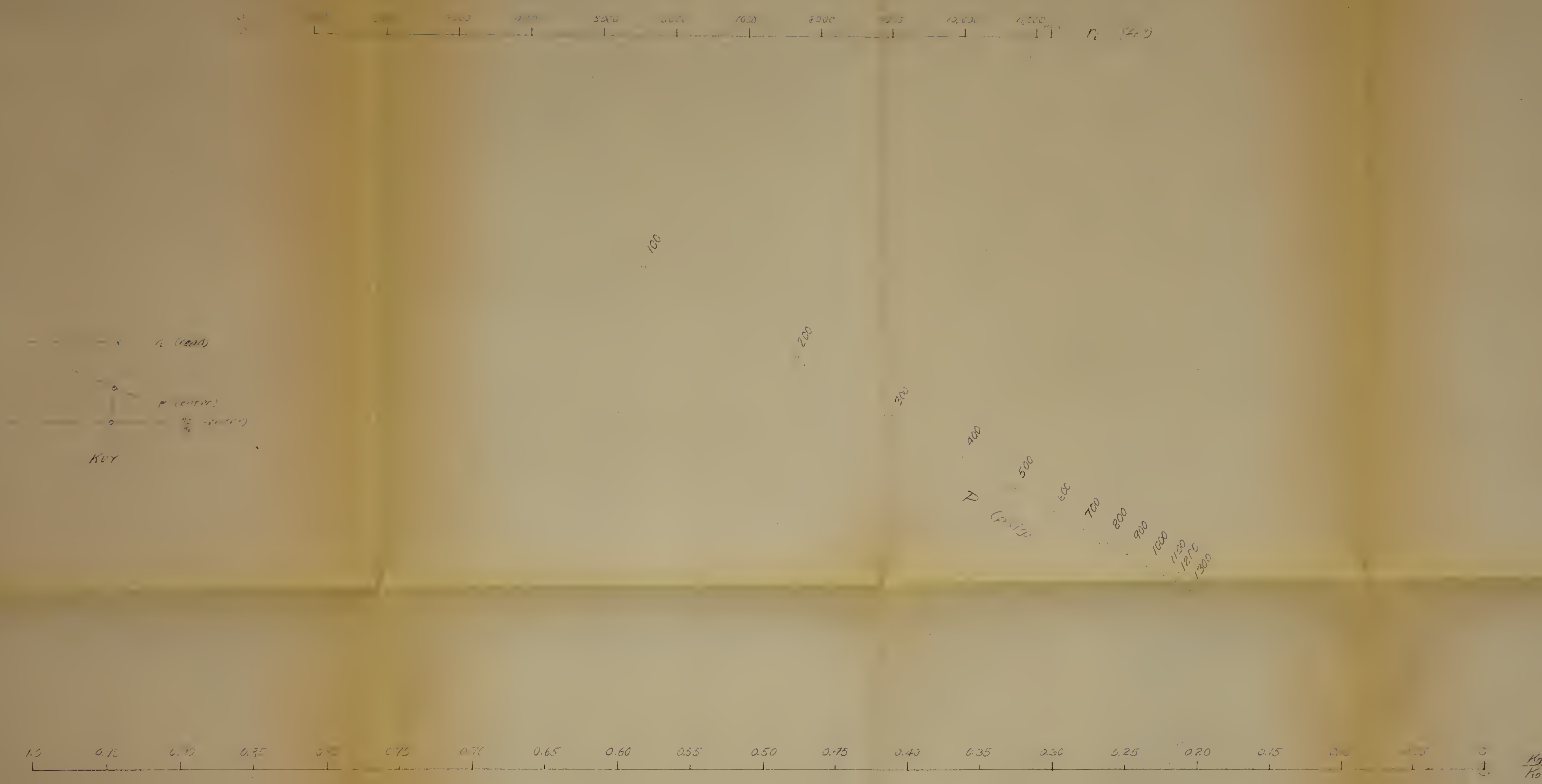
μ_g = Viscosity of produced gas, at reservoir pressure and temperature centipoises



Nomograph for Solution of Prediction Equation
n enter
G read
F enter
n
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1
G
0 50 100 150 200 250 300 350 400 450 500
P
1300 1200 1100 1000 900 800 700 600 500 400 300 200 100

LIQUID SATURAT[illegible] EQUATION FOR $\frac{K_g}{K_o}$ - S_L



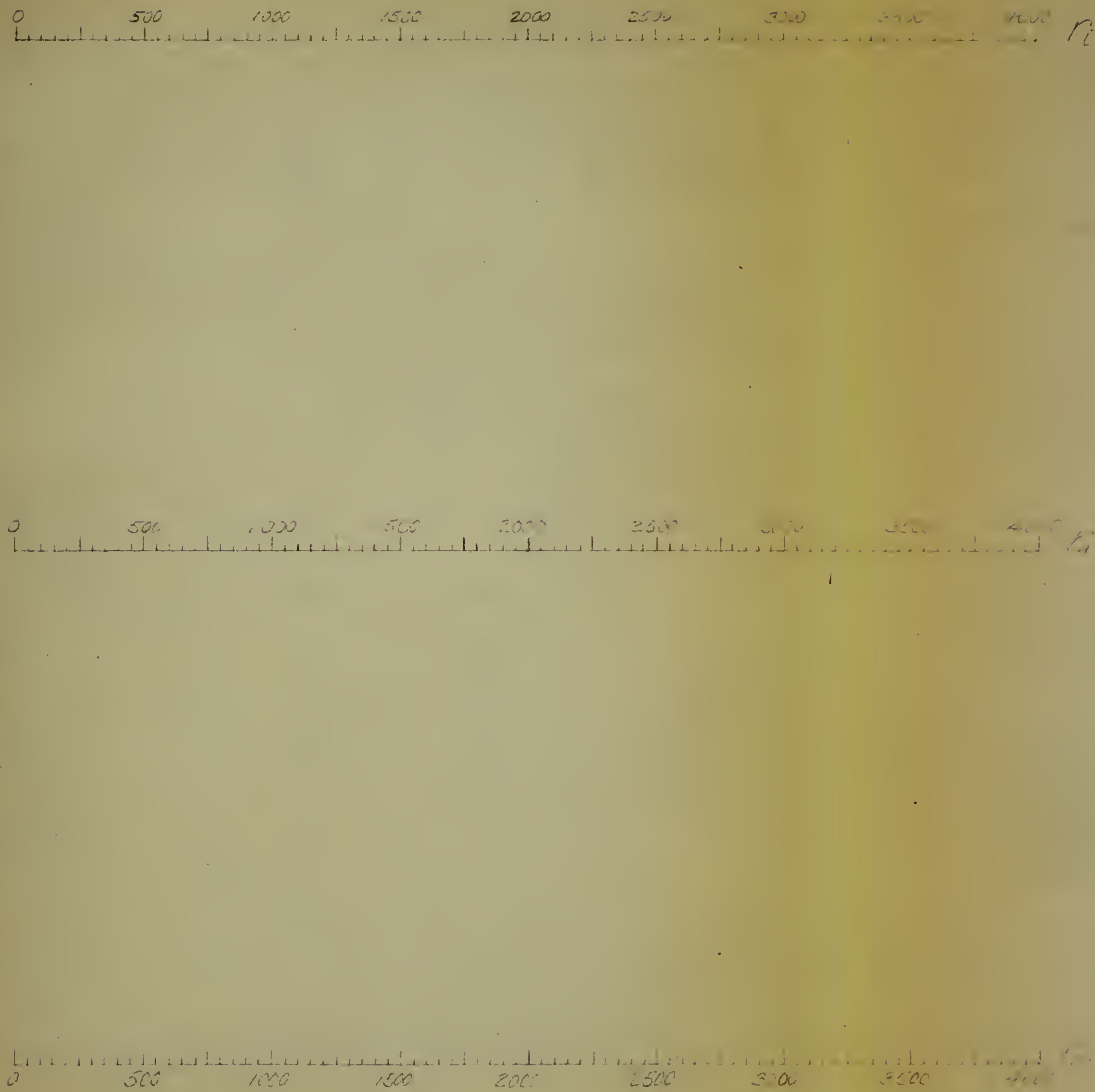


NOMOGRAPH FOR SOLUTION OF GAS-OIL RATIO EQUATION

$$r_i = \frac{\mu_o}{\mu_g}\frac{\beta}{V}\frac{K_g}{K_o} + S$$

0.2" = 0.001 unit $\frac{K_g}{K_o}$ 0.2" = 20 ft^3 of r_i

Nomograph for Solution of $\frac{r_o + r_i}{2} = r_a$



APPENDIX III

Considerations Affecting the Accuracy of Nomographs

1. Dimensions of the Diagram

The accuracy of a nomograph can be increased by increasing the dimensions of the diagram, within reasonable limits. This is due to the increase of relative accuracy and fineness of scale graduations.

2. Order of Accuracy

Nomographs, in general, have a degree of accuracy exceeding that of a slide rule of similar dimensions. In the nomograph, the ranges of the scales are tailored to the ranges of the variables involved. Short ranges are often expanded into long (and, therefore, more accurate) scales.

3. Decrease in Accuracy due to Complexity

The accuracy of a nomograph decreases sharply when more than one setting of a line must be made to obtain the answer. The more inter-related lines that must be drawn on a given nomograph to obtain a solution, the less accurate the answer will be.

4. Relative Placement of Scales

The accuracy of a nomograph is affected by the relative positions of the scales. If possible, the scale on which the answer is to be read should be between the other two scales.

APPENDIX IV

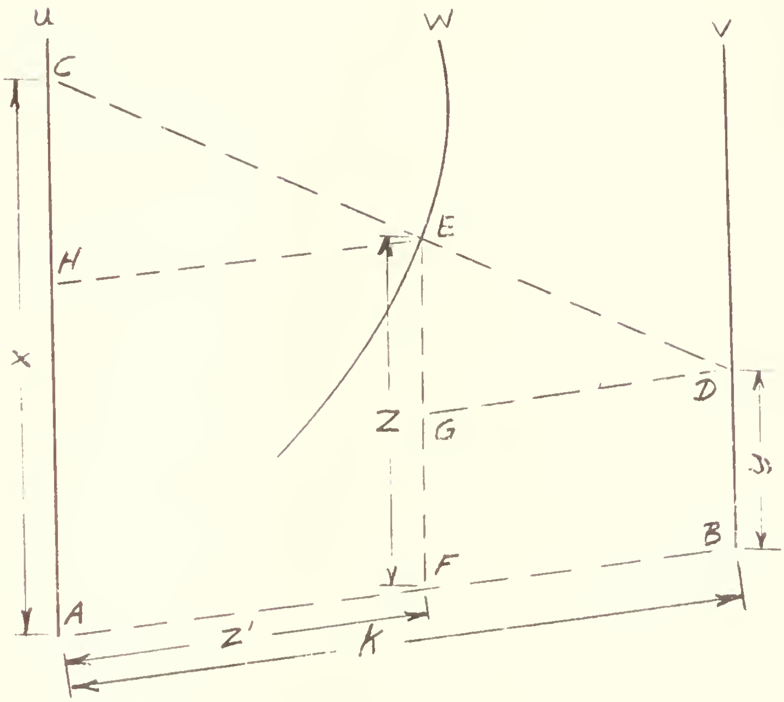
Geometric Proof for Nomographic Forms Used

A. Prediction Equation and Material Balance Equation

The equation form for the nomograph is:

$$f_1 (u) + f_2 (v) \times f_5 (W) = f_6 (W) \qquad (19)$$

Take two parallel axes for the variables u and v, and a curved axis in general for the variable W:



The line AB, of length k, joins the zero points of the scales for u and v. Points x and y are any points on the u and v scale, $x = m_1 f_1 (u)$ and $y = m_2 f_2 (v)$, where m_1 and m_2 are the respective scale moduli.

Take any index line CD intersecting the scale for W at E. Construct EF parallel to the u and v axes. Let EF = z and AF = z'. From similar triangles ECH and EDG:

$$\frac{x-z}{z'} = \frac{z-y}{k-z'}$$

$$(k-z')x = kz - zz' + zz' - yz'$$

$$(k-z')x + yz' = kz$$

$$x + \left(\frac{z'}{k-z'}\right)\left(y\right) = \left(\frac{k}{k-z'}\right)\left(z\right)$$

Substitute for x and y:

$$m_1 f_1 (u) + \left(\frac{z'}{k-z'}\right)\left[m_2 f_2 (v)\right] = \left(\frac{k}{k-z'}\right)\left(z\right)$$

$$f_1 (u) + \left(\frac{m_2}{m_1} \quad \frac{z'}{k-z'}\right)\left[f_2 (v)\right] = \frac{k}{m_1(k-z')}\left(z\right)$$

This equation will become the original equation if f_5 (W) and f_6 (W) have the following values:

$$f_5 (W) = \frac{m_2 z}{m_1(k-z')}$$

$$f_6 (W) = \frac{k_z}{m_1(k-z')}$$

Solving for z and z':

$$z = \frac{m_2 f_6 (W)}{f_5 (W) + \frac{m_2}{m_1}}$$

$$z' = \frac{k f_5 (W)}{f_5(W) + \frac{m_2}{m_1}}$$

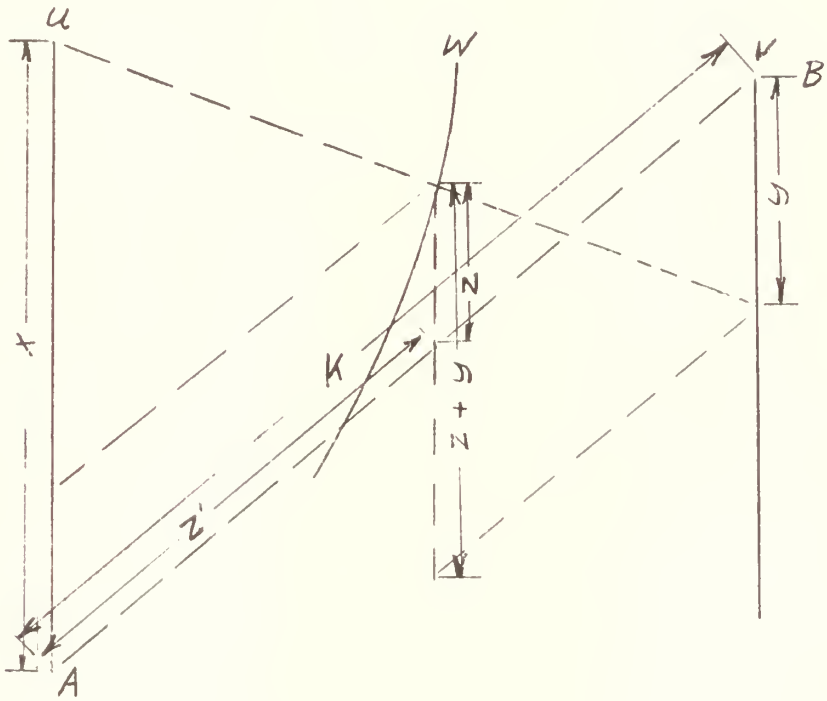
Points on the curve for the W-scale are obtained by taking different values of W and determining the corresponding values of z and z'. These points are plotted and a curve may be drawn through them.

B. Gas-Oil Ratio Equation

The form of the equation is:

$$f_1 (u) - f_2 (v) \cdot f_3 (W) = f_4 (W) \qquad (27)$$

This equation is similar in form to that discussed under "A Prediction Equation," and is subject to the same proof, except that the scale for f_2 (v) runs in the opposite direction from the f_1 (W) scale:



So that: $\frac{x-z}{z'} = \frac{z+y}{k-z'}$

After substitution for x and y, this reduces to:

$$f_1 (W) - \left(\frac{m_2}{m_1} \cdot \frac{z'}{k-z'}\right) \left[f_2 (v) \right] = \frac{k}{m_1 (k-z')}$$

and z and z' are as before:

$$z = \frac{m_2 f_4 (W)}{f_3 (W) + \frac{m_2}{m_1}}$$

$$z' = \frac{k f_3 (W)}{f_3 (W) + \frac{m_2}{m_1}}$$

Points on the W-scale are determined by taking different values of W and computing the corresponding values of z and z'.

C. The Liquid Saturation Equation

The final form of the liquid saturation equation was

$$\log (S_L - S_r) = \log \ldots$$ [illegible]

This is equivalent to [illegible]

[illegible]

[illegible]

let a, a, and [illegible]

[illegible]

Let ABC be a base line, perpendicular to the three scale axes, and let a and b be the distances between scales.

Draw any index line in general, FH, forming similar triangles DEF and DGH, where DE and GH are parallel to ABC.

From these similar triangles:

$$\frac{x - z}{a} = \frac{z - y}{b}$$

which rearranges to:

$$\frac{x}{a} + \frac{y}{b} = \frac{z}{ab/(a+b)}$$

Substituting particular functions for x, y, and z:

$$\frac{m_1 f_1 (u)}{a} + \frac{m_2 f_2 (v)}{b} = \frac{m_3 f_3 (W)}{ab/(a+b)}$$

To reduce this to the original form, let:

$$a = m_1$$

$$b = m_2$$

$$\frac{ab}{a + b} = m_3 \text{ , or } m_3 = \frac{m_1 m_2}{m_1 + m_2}$$

This also defines the scale moduli in terms of each other, showing that after m_1 and m_2 are selected, m_3 may be computed from them.

To find the position of the center scale in terms of distance between the outside scales, note that the distance from A to B in terms of AC is:

$$\frac{a}{a + b} \cdot (AC)$$

or

$$\frac{m_1}{m_1 + m_2} \cdot (AC)$$

D. Average Gas-Oil Ratio Equation

The equation form is:

$$r_i + r_{i-1} = 2\, r_a$$

or

$$f_1\,(u) + f_2\,(v) = f_3\,(W)$$

The proof is identical to that for the liquid saturation equation nomograph.

APPENDIX V

Data Concerning Reservoir "X"

A Mid-Continent Limestone Reservoir

Basic Data

Average Depth	5300 ft.
Original Reservoir Pressure	1773 psig
Reservoir Temperature	128° F.
Average Connate Water Saturation	33.4%
Original Formation Volume Factor, β_o	1.280
Original Solution Gas-Oil Ratio	500 $\frac{ft^3}{bbl.}$

P.V.T. Data

Pressure (psig)	S, (SCF/Bbl.STD)	β, F.V.F.	μ_o (C.P.)
1500	500	1.280	1.28
1250	448	1.260	1.32
1000	392	1.240	1.41
750	336	1.218	1.54
500	278	1.193	1.72
250	208	1.164	2.0
100	145	1.136	2.37
0	0	1.07	3.4

Volumetrically Weighted Average Bottom Hole Pressures

Date	BHP (Psig)	Cumulative Oil Production (Bbls.)
Original	1773	--
10-3-52	1654	152,330
4-13-53	1558	353,995
7-6-53	1517	443,999
10-1-53	1473	543,783
1-4-54	1441	631,764
7-6-54	1342	866,063
1-3-55	1209	1,086,797

Reservoir Fluid Characteristics:

Gravity of stock tank oil	41.5° API at 60° F.
Saturation pressure	1487 PSIG at 128° F.
Solution gas-oil ratio	484 Std. Cu.Ft./Resid. Bbl.
Formation volume factor	1.28 Res. Bbl./Resid. Bbl.
Viscosity of reservoir oil	1.28 Centipoise at 1487 PSI + 128°F.

Analysis of Reservoir Fluid Sample

	Weight %	Mol %
Methane and Lighter	3.39	26.35
Ethane	1.70	7.06
Propane	3.75	10.62
Iso-butane	0.82	1.77
N-butane	2.55	5.46
Iso-pentane	1.38	2.38
N-pentane	1.52	2.63
Hexanes	2.81	4.07
Heavier	82.08	39.66
	100.00	100.00

Gas Viscosity[18]

Pressure (Psig)	
1300	0.01477
1100	0.01401
900	0.01355
700	0.01280
500	0.01228
300	0.01193
100	0.01158

Reservoir Gas Analysis

Analysis of Gas Sample	Mol %
Methane	68.5
Ethane	8.0
Propane	7.6
N-Butane	2.5
Iso-Butane	1.2
N-Pentane	0.5
Iso-Pentane	0.8
Cyclo-Pentane	0.2
Hexanes plus	0.4
Nitrogen	9.9
Oxygen	0.1
Helium	0.1
Hydrogen	0.1
CO_2	0.1
H_2S	0.1
	100.00

Gas gravity at 60°F. + 14.7 psi = 0.819 (Air = 1.000)

Relative Permeability Data

S_L (%)	$\frac{K_g}{K_o}$
98	0.01
97	0.0181
96	0.033
95	0.057
90	0.373
85	1.15
80	3.20
75	8.0

Summary of Production:

Year	Producing Wells at End of Year	Production Oil (Bbls.) Year	Production Oil (Bbls.) Cumulative	Production Gas (MCF) Year	Production Gas (MCF) Cumulative
1951	5	27,295	27,295	13,655	13,655
1952	26	185,930	213,225	93,105	106,760
1953	29	414,034	627,259	254,261	361,021
1954	31	456,238	1,083,497	753,605	1,114,626

Production prior to bubble point: 575,000 bbls. STO

Total Volume of Original Oil in Place:

$N = 18,000,000$ Bbls. STO (Calculated by Material Balance and verified by volumetric value)

On Basis of $N = 1$ Bbl. STO,

Oil Production, B.P. to 1209 psig

$n = 0.0319$ Bbl.

Gas Production, BP to 1209 psig

$G = 46.75$ SCF

Instantaneous Gas-Oil Ratio at 1209 psig

$r_i = 2400$ SCF/Bbl.

APPENDIX VI

Prediction of Performance of Reservoir "X" by Nomographic Method

Line No.	P	(Historical Point) 1209	1000	800	600	400	300	200	100
(1)	n		0.0135	0.010	0.00745	0.00733	0.00376	0.00462	0.00684
(2)	n_{i-1}		0.032	0.0455	0.0545	0.06195	0.06928	0.07304	0.07766
(3)	n_i	0.0319	0.0455	0.0545	0.06195	0.06928	0.07304	0.07766	0.08450
(4)	$G_{i-1} + r_a \quad n$	46.75	85.90	131.9	181.1	239.1	270.9	308.7	358.0
(5)	G_{i-1}		46.75	85.9	131.9	181.1	239.1	270.9	308.7
(6)	$r_a \quad n$		39.15	46.0	49.2	58.0	31.8	37.8	49.3
(7)	r_a		2900	4600	6600	7910	8450	8180	7210
(8)	r_{i-1}		2400	3400	5800	7380	8450	8450	7910
(9)	r_i	2400	3400	5800	7380	8450	8450	7910	6510
(10)	$\frac{K_g}{K_o}$		0.057	0.124	0.194	0.305	0.384	0.494	0.692
(11)	r_i (from G.O.R.)		3400	5820	7380	8450	8450	7910	6520
(12)	(n_i) (1.8×10^7)		819.5	981	1,116	1,249	1,317	1,399	1,521
(13)	+575,000		575	575	575	575	575	575	575
(14)	Cumulative Production (in 10^3 bbls.)		1,394.5	1,556	1,691	1,824	1,892	1,974	2,096

Prediction of Performance of Reservoir "X" by the Nomographic Method
(Using Alternate Method)

Line No.	P	1209 (A known point of Production History)	1100	900	700	500	300	400	300	100
(1)	n		0.00773	0.01248	0.00845	0.00665	0.00741	0.00360	0.00372	0.01145
(2)	n_{i-1}		0.0319	0.03963	0.05211	0.06056	0.06721	0.06721	0.07081	0.07453
(3)	n_i	0.0319	0.03963	0.05211	0.06056	0.06721	0.07462	0.07081	0.07453	0.08598
(4)	G_{i-1} + n r_a	46.75	65.5	108.3	156.8	208.5	271.2	239.2	271.2	358.0
(5)	G_{i-1}		46.75	65.5	108.3	156.8	208.5	208.5	239.2	271.2
(6)	r_a n		18.75	42.8	48.5	51.7	62.7	30.7	32.0	86.8
(7)	r_a		2425	3430	5745	7720	8465	8525	8615	7575
(8)	r_{i-1}		2400	2450	4420	7070	8370	8560	8680	8550
(9)	r_i	2400	2450	4420	7070	8370	8560	8680	8550	6600
(10)	$\frac{K_g}{K_o}$		0.0362	0.085	0.1658	0.253	0.390	0.314	0.390	0.70
(11)	r_i (from G.O.R.)		2450	4420	7070	8370	8460	8680	8550	6590
(12)	(n_i) (1.8×10^7)		713.0	937.5	1090.5	1210.0	1342.0	1276.0	1341.0	1549.0
(13)	+575,000		575.0	575.0	575.0	575.0	575.0	575.0	575.0	575.0
(14)	Cumulative Production (in 10^3 bbls.)		1288.0	1512.5	1665.5	1785.0	1917.0	1851.0	1916.0	2124

Calculations for Prediction Equation

(1) P	(2) $f_5(W)$ $\left(\frac{\phi g}{\phi n}\right)$	(3) $f_6(W)$ $\left(\frac{1}{\phi n}\right)$	(4) $k.f_5(W)$	(5) $f_5(W)+\frac{m_2}{m_1}$	(6) $m_2 f_6(W)$	(7) $z''=\frac{(4)}{(5)}$	(8) $z = \frac{(6)}{(5)}$
1300	0.004207	0.1396	0.054691	0.004607	0.005584	11.871	1.212
1200	0.005150	0.2623	0.06695	0.005550	0.010492	12.063	1.890
1100	0.006502	0.4644	0.084526	0.006902	0.018576	12.247	2.691
1000	0.008888	0.8064	0.115544	0.009238	0.032256	12.440	3.473
900	0.01306	1.471	0.16978	0.01346	0.05884	12.614	4.372
800	0.02435	3.278	0.31655	0.02475	0.13112	12.790	5.298
700	0.12139	19.135	1.57807	0.12179	0.76540	12.957	6.285
600	(-)0.04390	(-)7.918	(-)0.57070	0.04350	(-)0.31672	13.120	7.281
500	(-)0.01985	(-)4.078	(-)0.25805	0.01945	(-)0.16312	13.267	8.387
400	(-)0.01392	(-)3.262	(-)0.18096	0.01352	(-)0.13048	13.385	9.651
300	(-)0.0110	(-)2.911	(-)0.1430	0.01060	(-)0.11644	13.491	10.985
200	(-)0.009848	(-)2.971	(-)0.128024	0.009448	(-)0.11884	13.550	12.578
100	(-)0.01010	(-)3.538	(-)0.13120	0.00970	(-)0.14152	13.536	14.590

$$z' = \frac{k\, f_5(W)}{f_5(W) + \frac{m_2}{m_1}}$$

$$z = \frac{m_2\, f_6(W)}{f_5(W) + \frac{m_2}{m_1}}$$

$k = 13''$

$m_1 = 100$ (scale = 10")

$m_2 = 0.04$ (scale = 20")

$$\frac{m_2}{m_1} = 0.0004$$

Scale Divisions:

n: $1'' = 0.01$ unit of n

$\frac{1}{50}'' = 0.0002$ unit n

g: $1'' = 25$ units of g

$\frac{1}{50}'' = 0.5$ unit of g

Calculation of $f_5(W) = \frac{1}{\frac{\beta}{v} - s}$ and $f_6(W) = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)}$

P	f_5 (W)	f_6 (W)
1300	0.004207	0.1396
1200	0.005150	0.2623
1100	0.006502	0.4644
1000	0.00888	0.8064
900	0.01306	1.471
800	0.02435	3.278
700	0.12139	19.135
600	0.04390	(-)7.918
500	0.01985	(-)4.078
400	0.01392	(-)3.262
300	0.01100	(-)2.911
200	0.009848	(-)2.971
100	0.01010	(-)3.538

Calculations for Nomograph to Solve Gas-Oil Ratio Equation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	P (W)	F f_3(W)	S f_4(W)	K.F Kf_3(W)	$F + \frac{m_2}{m_1}$	M_2S m_2f_4(W)	$z' = \frac{(4)}{(5)}$	$z = \frac{(6)}{(5)}$
Moduli	1300	61935.057	458	1238701.140	81935.057	9160	15.118	0.111
$\frac{K_g}{K_o}$ (or $f_2(v)$; m_2 = 20	1200	58189.950	437	1163799.00	78189.950	8740	14.884	0.111
	1100	55522.293	414	1110445.860	75522.293	8280	14.703	0.109
r_i (or f_1(W); m_1] 0.001	1000	51724.011	393	1034480.220	71724.011	7860	14.423	0.109
Plotting Equations for	900	47949.257	370	958985.140	67949.257	7400	14.113	0.108
$\frac{K_g}{K_o}$ and r_i	800	44391.712	347	887834.240	64391.712	6940	13.788	0.107
	700	40751.027	324	815020.54	60751.027	6480	13.415	0.106
$\frac{K_g}{K_o}$: Dist.=(20) $(\frac{K_g}{K_o})$	600	36604.360	302	732087.20	56604.360	6040	12.933	0.106
r_i] Dist.=(0.001)(r_i)	500	31975.386	278	639507.72	51975.386	5560	12.304	0.106
Scale Lengths:	400	26837.195	251	536743.90	46837.195	5020	11.459	0.107
	300	21364.723	223	427294.46	41364.723	4460	10.329	0.107
$\frac{K_g}{K_o}$: 18" length to zero = 11.2"	200	15548.990	189	310979.80	35548.990	3780	8.747	0.106
k = 20"	100	9235.955	144	184719.10	29235.955	2880	6.318	0.098

Computation of Scale Lengths for Liquid Saturation Equation

$$\text{Log } (S_L - K_1) = \text{Log } (K_2 \beta) + \text{Log } (1-n_i)$$

$$K_1 = S_W = 0.334 \qquad K_2 = \frac{(1-S_W)}{\beta_o} = \frac{0.666}{1.280} = 0.5203$$

	Scale Label	Labeled Limits	Variable	Limits	Function	Range of Function	Scale Length	Actual Scale Modulus	Plotting Modulus	Scale Equation
	P	1300 to 100		1.264 1.135 0.129	log K_2	9.81803-10 9.77122-10 0.04681	14.043"	300	300	$x=300 \log K_2\beta$
12" apart	n	0.03 0.10	n_i	$1-n_i$ 0.97 0.90	$\log(1-n_i)$	9.98677-10 9.95424-10 0.03253	9.759"	300	300	$y=300 \log(1-n_i)$
6" from others	S_L		S_L		$\log(S_L-K_1)$			150	150	

(1)	(2)	(3)	(4)	(5)	(6)
P(psig)	β	$K_2\beta$	Log $K_2\beta$	300Xlog$K_2\beta$	(5)-*
1300	1.264	0.6577	9.81803-10	2945.409	14.043
1200	1.256	0.6535	9.81525	2944.575	13.209
1100	1.248	0.6493	9.81245	2943.735	12.369
1000	1.240	0.6452	9.80969	2942.907	11.541
900	1.232	0.6410	9.80686	2942.058	10.692
800	1.222	0.6358	9.80332	2940.996	9.630
700	1.213	0.6311	9.80010	2940.030	8.664
600	1.204	0.6264	9.79685	2939.055	7.689
500	1.193	0.6207	9.79288	2937.864	6.498
400	1.183	0.6155	9.78923	2936.769	5.403
300	1.171	0.6093	9.78483	2935.449	4.083
200	1.156	0.6015	9.77924	2933.772	2.406
100	1.135	0.5905	9.77122-10	2931.366*	0

(1)	(2)	(3)	(4)	(5)
n	(1-n)	log(1-n)	300X(3)	(4)-*
0.03	0.970	9.98677-10	2996.031	9.759
0.035	0.965	9.98453	2995.359	9.087
0.040	0.960	9.98227	2994.681	8.409
0.045	0.955	9.98000	2994.000	7.728
0.050	0.950	9.97772	2993.316	7.044
0.055	0.945	9.97543	2992.629	6.357
0.060	0.940	9.97313	2991.939	5.667
0.065	0.935	9.97081	2991.243	4.971
0.070	0.930	9.968.48	2990.544	4.272
0.075	0.925	9.96614	2989.842	3.570
0.080	0.920	9.96379	2989.137	2.865
0.085	0.915	9.96142	2988.426	2.154
0.090	0.910	9.95904	2987.712	1.440
0.95	0.905	9.95665	2986.995	0.723
0.10	0.90	9.95424	2986.272*	0

Limiting Values of S_L:

P = 1300; n_i = 0.03

P = 100; n_i = 0.10

Using

$$S_L = S_W + (1-S_W)\,\frac{(1-n_i)}{\beta_o}\,\beta$$

(1)	(2)	(3)	(4)	Limit Values of				
$\frac{1-S_W}{\beta_o}$	(1-n)	(1)x(2)	(3)+S_W=S_L	S_L-K_1	LogS_L-K_1	Log x 150	Actual Log S_L-K_1	Actual Log x150
0.5203	1.226	0.6379	0.9719	0.6379	9.80480-10	1470.720	-0.19520	-29.280
0.5203	1.0215	0.5315	0.8655	0.5315	9.72546-10	1458.819	-0.27454	-41.181
						11.901 Scale Length		-11.901 Diff. = Scale Length

Liquid Saturation Equation Nomograph - Computation of S_L Scale Values

Scale Modulus, $m_{S_L} = 150$; Scale Length = 11.901 inches

Scale Equation = $z = (150)(\log S_L - K_1)$, $K_1 = 0.334$

For Column (6) subtract $(150)(\text{Log } S_L - K_1)$ for T = 100, $n_i = 0.1$. This value is 1458.819 - 1500

(1) K_g/K_o	(2) S_L	(3) $S_L - K_1$	(4) $\text{Log}(S_L - K_1)$	(5) 150x(4)	(6) (5) Zero Reading	
0.01	0.980	0.646	9.81023-10	1471.534	12.715	Plotting values for K_g/K_o side of scale
0.02	0.968	0.634	9.80209	1470.314	11.495	
0.03	0.961	0.627	9.79727	1469.590	10.771	
0.04	0.956	0.622	9.79379	1469.068	10.249	
0.05	0.952	0.618	9.79099	1468.648	9.829	
0.06	0.949	0.615	9.78888	1468.332	9.513	
0.07	0.946	0.612	9.78675	1468.012	9.193	
0.08	0.943	0.609	9.78462	1467.693	8.874	
0.09	0.9405	0.6065	9.78283	1467.424	8.605	
0.10	0.939	0.605	9.78176	1467.264	8.445	
0.15	0.9302	0.5962	9.77539	1466.308	7.489	
0.20	0.9205	0.5865	9.76827	1465.240	6.421	
0.25	0.9135	0.5795	9.76305	1464.458	5.639	
0.30	0.9075	0.5735	9.75853	1463.780	4.961	
0.40	0.897	0.563	9.75051	1462.576	3.757	
0.50	0.888	0.554	9.74351	1461.526	2.707	
0.60	0.881	0.547	9.73799	1460.698	1.879	
0.70	0.874	0.540	9.73239	1459.858	1.039	
0.80	0.867	0.533	9.72673	1459.010	0.191	
0.90	0.861	0.527	9.72181	1458.272	-0.547	
1.00	0.856	0.522	9.71767	1457.650	-1.169	
	0.97	0.636	9.8035-10	1470.525	11.706	Plotting values for S_L side of scale
	0.96	0.626	9.7966	1469.490	10.671	
	0.95	0.616	9.7896	1468.440	9.621	
	0.94	0.606	9.7825	1467.375	8.556	
	0.93	0.596	9.7752	1466.280	7.461	
	0.92	0.586	9.7679	1465.185	6.366	
	0.91	0.576	9.7604	1464.060	5.241	
	0.90	0.566	9.7528	1462.920	4.101	
	0.89	0.556	9.7451	1461.765	2.446	
	0.88	0.546	9.7372	1460.580	1.761	
	0.87	0.536	9.7292	1459.380	0.561	

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Thesis 32844
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